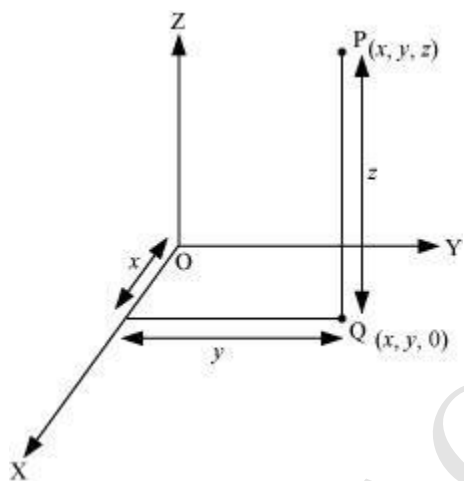


Introduction to Three Dimensional Geometry

- **Three-dimensions coordinate planes**

- The coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called x , y , and z -axes.
- The three planes determined by the pair of axes are the coordinate planes, called XY , YZ and ZX -planes.
- The three coordinate planes divide the space into eight parts known as octants.
- In three-dimensional geometry, the coordinates of a point P are always written in the form of triplets i.e., (x, y, z) . Here, x , y , and z are the distances from the YZ , ZX and XY -planes. Also, the coordinates of the origin are $(0, 0, 0)$.



- The sign of the coordinates of a point determine the octant in which the point lies. The following table shows the signs of the coordinates in the eight octants.

Oc tan ts → Coordinates ↓	I	II	III	IV	V	VI	VII	VIII
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

Example: The point $(-5, 6, -7)$ lies in the VI octant.

- In Coordinates of points lying on different axes:
 - Any point on the x -axis is of the form $(x, 0, 0)$
 - Any point on the y -axis is of the form $(0, y, 0)$

- Any point on the z-axis is of the form $(0, 0, z)$
- Coordinates of points lying in different planes:
 - Coordinates of a point in the YZ-plane are of the form $(0, y, z)$
 - Coordinates of a point in the XY-plane are of the form $(x, y, 0)$
 - Coordinates of a point in the ZX-plane are of the form $(x, 0, z)$

Example: The points $(-5, 6, 0)$, $(0, -5, 6)$, $(-5, 0, 6)$ lies in the XY-plane, YZ-plane and ZX-plane respectively.

- **distance formula** Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example: Find the point(s), lying on the z-axis, whose distance from point $(2, -1, 3)$ is 3 units.

Solution: Let the required point be $(0, 0, z)$. We know that the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Therefore,

$$\sqrt{(2 - 0)^2 + (-1 - 0)^2 + (3 - z)^2} = 3$$

On squaring both the sides, we get

$$4 + 1 + 9 + z^2 - 6z = 9$$

$$\Rightarrow z^2 - 6z + 5 = 0$$

$$\Rightarrow z^2 - 5z - z + 5 = 0$$

$$\Rightarrow z(z - 5) - 1(z - 5) = 0$$

$$\Rightarrow z = 1, 5$$

Thus, the required points on the z-axis are $(0, 0, 1)$ and $(0, 0, 5)$.